

# Pari-GP reference card

(PARI-GP version 2.11.0)

Note: optional arguments are surrounded by braces {}.

To start the calculator, type its name in the terminal: **gp**

To exit **gp**, type **quit**, **\q**, or **<C-D>** at prompt.

## Help

describe function	?function
extended description	??keyword
list of relevant help topics	???pattern
name of GP-1.39 function $f$ in GP-2.*	whatnow( $f$ )

## Input/Output

previous result, the result before	%, %', %'', etc.
$n$ -th result since startup	% $n$
separate multiple statements on line	;
extend statement on additional lines	\
extend statements on several lines	{seq <sub>1</sub> ; seq <sub>2</sub> ;}
comment	/* ... */
one-line comment, rest of line ignored	\\ ...

## Metacommands & Defaults

set default $d$ to $val$	default({ $d$ },{ $val$ })
toggle timer on/off	#
print time for last result	##
print defaults	\d
set debug level to $n$	\g $n$
set memory debug level to $n$	\gm $n$
set $n$ significant digits / bits	\p $n$ , \pb $n$
set $n$ terms in series	\ps $n$
quit GP	\q
print the list of PARI types	\t
print the list of user-defined functions	\u
read file into GP	\r filename

## Debugger / break loop

get out of break loop	break or <C-D>
go up/down $n$ frames	dbg_up({ $n$ }), dbg_down
set break point	breakpoint()
examine object $o$	dbg_x( $o$ )
current error data	dbg_err()
number of objects on heap and their size	getheap()
total size of objects on PARI stack	getstack()

## PARI Types & Input Formats

<b>t_INT</b> . Integers; hex, binary	$\pm 31$ ; $\pm 0x1F$ , $\pm 0b101$
<b>t_REAL</b> . Reals	$\pm 3.14$ , 6.022 E23
<b>t_INTMOD</b> . Integers modulo $m$	Mod( $n, m$ )
<b>t_FRAC</b> . Rational Numbers	$n/m$
<b>t_FFELT</b> . Elt in finite field $\mathbf{F}_q$	ffgen( $q$ , 't)
<b>t_COMPLEX</b> . Complex Numbers	$x + y * I$
<b>t_PADIC</b> . $p$ -adic Numbers	$x + O(p^k)$
<b>t_QUAD</b> . Quadratic Numbers	$x + y * \text{quadgen}(D, 'w)$
<b>t_POLMOD</b> . Polynomials modulo $g$	Mod( $f, g$ )
<b>t_POL</b> . Polynomials	$a * x^n + \dots + b$
<b>t_SER</b> . Power Series	$f + O(x^k)$
<b>t_RFRAC</b> . Rational Functions	$f/g$
<b>t_QFI</b> / <b>t_QFR</b> . Imag/Real binary quad. form	Qfb( $a, b, c, \{d\}$ )
<b>t_VEC</b> / <b>t_COL</b> . Row/Column Vectors	$[x, y, z]$ , $[x, y, z] \sim$
<b>t_VEC</b> integer range	$[1..10]$

<b>t_VECSMALL</b> . Vector of small ints	Vecsmall( $[x, y, z]$ )
<b>t_MAT</b> . Matrices	$[a, b; c, d]$
<b>t_LIST</b> . Lists	List( $[x, y, z]$ )
<b>t_STR</b> . Strings	"abc"
<b>t_INFINITY</b> . $\pm\infty$	+oo, -oo

## Reserved Variable Names

$\pi = 3.14\dots$ , $\gamma = 0.57\dots$ , $C = 0.91\dots$	Pi, Euler, Catalan
square root of $-1$	I
Landau's big-oh notation	O

## Information about an Object

PARI type of object $x$	type( $x$ )
length of $x$ / size of $x$ in memory	# $x$ , sizebyte( $x$ )
real precision / bit precision of $x$	precision( $x$ ), bitprecision
$p$ -adic, series prec. of $x$	padicprec( $x$ ), serprec

## Operators

basic operations	+, -, *, /, ^, sqr
$i=i+1$ , $i=i-1$ , $i=i*j$ , ...	i++, i--, i*=j,...
eulidean quotient, remainder	$x \backslash y$ , $x \backslash y$ , $x \backslash y$ , divrem( $x, y$ )
shift $x$ left or right $n$ bits	$x << n$ , $x >> n$ or shift( $x, \pm n$ )
multiply by $2^n$	shiftmul( $x, n$ )
comparison operators	<=, <, >=, >, ==, !=, ==, lex, cmp
boolean operators (or, and, not)	, &&, !
bit operations	bitand, bitneg, bitor, bitxor, bitneginv
maximum/minimum of $x$ and $y$	max, min( $x, y$ )
sign of $x = -1, 0, 1$	sign( $x$ )
binary exponent of $x$	exponent( $x$ )
derivative of $f$	$f'$
differential operator	diffop( $f, v, d, \{n = 1\}$ )
quote operator (formal variable)	'x
assignment	x = value
simultaneous assignment $x \leftarrow v_1, y \leftarrow v_2$	[x,y] = v

## Select Components

$n$ -th component of $x$	component( $x, n$ )
$n$ -th component of vector/list $x$	$x[n]$
components $a, a + 1, \dots, b$ of vector $x$	$x[a..b]$
$(m, n)$ -th component of matrix $x$	$x[m, n]$
row $m$ or column $n$ of matrix $x$	$x[m, ]$ , $x[, n]$
numerator/denominator of $x$	numerator( $x$ ), denominator

## Random Numbers

random integer/prime in $[0, N[$	random( $N$ ), randomprime
get/set random seed	getrand, setrand( $s$ )

## Conversions

to vector, matrix, vec. of small ints	Col/Vec, Mat, Vecsmall
to list, set, map, string	List, Set, Map, Str
create PARI object ( $x \bmod y$ )	Mod( $x, y$ )
make $x$ a polynomial of $v$	Pol( $x, \{v\}$ )
as Pol, etc., starting with constant term	Polrev, Vecrev, Colrev
make $x$ a power series of $v$	Ser( $x, \{v\}$ )
string from bytes / from format+args	Strchr, Strprintf
TeX string	Strtex( $x$ )
convert $x$ to simplest possible type	simplify( $x$ )
object $x$ with real precision $n$	precision( $x, n$ )
object $x$ with bit precision $n$	bitprecision( $x, n$ )
set precision to $p$ digits in dynamic scope	localprec( $p$ )
set precision to $p$ bits in dynamic scope	localbitprec( $p$ )

## Conjugates and Lifts

conjugate of a number $x$	conj( $x$ )
norm of $x$ , product with conjugate	norm( $x$ )
$L^p$ norm of $x$ ( $L^\infty$ if no $p$ )	normlp( $x, \{p\}$ )
square of $L^2$ norm of $x$	norml2( $x$ )
lift of $x$ from Mods and $p$ -adics	lift, centerlift( $x$ )
recursive lift	liftall
lift all <b>t_INT</b> and <b>t_PADIC</b> ( $\rightarrow$ <b>t_INT</b> )	liftint
lift all <b>t_POLMOD</b> ( $\rightarrow$ <b>t_POL</b> )	lifttpol

## Lists, Sets & Maps

**Sets** (= row vector with strictly increasing entries w.r.t. cmp)

intersection of sets $x$ and $y$	setintersect( $x, y$ )
set of elements in $x$ not belonging to $y$	setminus( $x, y$ )
union of sets $x$ and $y$	setunion( $x, y$ )
does $y$ belong to the set $x$	setsearch( $x, y, \{flag\}$ )
set of all $f(x, y)$ , $x \in X$ , $y \in Y$	setbinop( $f, X, Y$ )
is $x$ a set ?	setisset( $x$ )

**Lists**. create empty list:  $L = \text{List}()$

append $x$ to list $L$	listput( $L, x, \{i\}$ )
remove $i$ -th component from list $L$	listpop( $L, \{i\}$ )
insert $x$ in list $L$ at position $i$	listinsert( $L, x, i$ )
sort the list $L$ in place	listsort( $L, \{flag\}$ )

**Maps**. create empty dictionary:  $M = \text{Map}()$

attach value $v$ to key $k$	mapput( $M, k, v$ )
recover value attach to key $k$ or error	mapget( $M, k$ )
is key $k$ in the dict ? (set $v$ to $M(k)$ )	mapisdefined( $M, k, \{\&v\}$ )
remove $k$ from map domain	mapdelete( $M, k$ )

## GP Programming

### User functions and closures

$x, y$  are formal parameters;  $y$  defaults to Pi if parameter omitted;  
 $z, t$  are local variables (lexical scope),  $z$  initialized to 1.

```
fun(x, y=Pi) = my(z=1, t); seq
fun = (x, y=Pi) -> my(z=1, t); seq
```

attach a help message to $f$	addhelp( $f$ )
undefine symbol $s$ (also kills help)	kill( $s$ )
<b>Control Statements</b> ( $X$ : formal parameter in expression $seq$ )	
if $a \neq 0$ , evaluate $seq_1$ , else $seq_2$	if( $a, \{seq_1\}, \{seq_2\}$ )

eval. $seq$ for $a \leq X \leq b$	for( $X = a, b, seq$ )
...for primes $a \leq X \leq b$	forprime( $X = a, b, seq$ )
...for primes $\equiv a \pmod{q}$	forprimestep( $X = a, b, q, seq$ )
...for composites $a \leq X \leq b$	forcomposite( $X = a, b, seq$ )
...for $a \leq X \leq b$ stepping $s$	forstep( $X = a, b, s, seq$ )
...for $X$ dividing $n$	fordiv( $n, X, seq$ )
... $X = [n, factor(n)]$ , $a \leq n \leq b$	forfactored( $X = a, b, seq$ )
...as above, $n$ squarefree	forsquarefree( $X = a, b, seq$ )
... $X = [d, factor(d)]$ , $d \mid n$	fordivfactored( $n, X, seq$ )
multivariable for, lex ordering	forvec( $X = v, seq$ )
loop over partitions of $n$	forpart( $p = n, seq$ )
...permutations of $S$	forperm( $S, p, seq$ )
...subsets of $\{1, \dots, n\}$	forsubset( $n, p, seq$ )
... $k$ -subsets of $\{1, \dots, n\}$	forsubset( $[n, k], p, seq$ )
...vectors $v$ , $q(v) \leq B$ ; $q > 0$	forqfvec( $v, q, b, seq$ )
... $H < G$ finite abelian group	forsubgroup( $H = G$ )

evaluate $seq$ until $a \neq 0$	until( $a, seq$ )
while $a \neq 0$ , evaluate $seq$	while( $a, seq$ )
exit $n$ innermost enclosing loops	break({ $n$ })
start new iteration of $n$ -th enclosing loop	next({ $n$ })
return $x$ from current subroutine	return({ $x$ })

## Exceptions, warnings

raise an exception / warn

type of error message  $E$

try  $seq_1$ , evaluate  $seq_2$  on error

## Functions with closure arguments / results

select from  $v$  according to  $f$

apply  $f$  to all entries in  $v$

evaluate  $f(a_1, \dots, a_n)$

evaluate  $f(\dots f(f(a_1, a_2), a_3) \dots, a_n)$

calling function as closure

## Sums & Products

sum  $X = a$  to  $X = b$ , initialized at  $x$

sum entries of vector  $v$

product of all vector entries

sum  $expr$  over divisors of  $n$

... assuming  $expr$  multiplicative

product  $a \leq X \leq b$ , initialized at  $x$

product over primes  $a \leq X \leq b$

## Sorting

sort  $x$  by  $k$ -th component

min.  $m$  of  $x$  ( $m = x[i]$ ), max.

does  $y$  belong to  $x$ , sorted wrt.  $f$

## Input/Output

print with/without  $\backslash n$ ,  $\text{\TeX}$  format

pretty print matrix

print fields with separator

formatted printing

write  $args$  to file

write  $x$  in binary format

read file into GP

... return as vector of lines

... return as vector of strings

read a string from keyboard

## Files and file descriptors

File descriptors allows efficient small consecutive reads or writes from or to a given file. The argument  $n$  below is always a descriptor, attached to a file in **r**(ead), **w**(rite) or **a**(ppend) mode.

get descriptor  $n$  for file  $path$  in given  $mode$   
... from shell  $cmd$  output (pipe)

close descriptor

commit pending write operations

read logical line from file

... raw line from file

write  $s \backslash n$  to file

... write  $s$  to file

## Timers

CPU time in  $ms$  and reset timer

CPU time in  $ms$  since gp startup

time in  $ms$  since UNIX Epoch

timeout command after  $s$  seconds

## Interface with system

allocates a new stack of  $s$  bytes

alias  $old$  to  $new$

install function from library

execute system command  $a$

... and feed result to GP

... returning GP string

**error()**, **warning()**

**errname( $E$ )**

**iferr( $seq_1, E, seq_2$ )**

**select( $f, v$ )**

**apply( $f, v$ )**

**call( $f, a$ )**

**fold( $f, a$ )**

**self()**

**sum( $X = a, b, expr, \{x\}$ )**

**vecsum( $v$ )**

**vecprod( $v$ )**

**sumdiv( $n, X, expr$ )**

**sumdivmult( $n, X, expr$ )**

**prod( $X = a, b, expr, \{x\}$ )**

**prodeuler( $X = a, b, expr$ )**

**vecsrt( $x, \{k\}, \{fl = 0\}$ )**

**vecmin( $x, \{\&i\}$ ), **vecmax****

**vecsearch( $x, y, \{f\}$ )**

**print, print1, printtex**

**printp**

**printsep( $sep, \dots$ ), **printsep1****

**printf()**

**write, write1, writetex( $file, args$ )**

**writebin( $file, x$ )**

**read( $\{file\}$ )**

**readvec( $\{file\}$ )**

**readstr( $\{file\}$ )**

**input()**

**fileopen( $path, mode$ )**

**fileextern( $cmd$ )**

**fileclose( $n$ )**

**fileflush( $n$ )**

**fileread( $n$ )**

**filereadstr( $n$ )**

**filewrite( $n, s$ )**

**filewrite1( $n, s$ )**

**gettime()**

**getabstime()**

**getwalltime()**

**alarm( $s, expr$ )**

**allocatemem( $\{s\}$ )**

**alias( $new, old$ )**

**install( $f, code, \{gpf\}, \{lib\}$ )**

**system( $a$ )**

**extern( $a$ )**

**externstr( $a$ )**

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get  $\$VAR$  from environment

expand env. variable in string

**getenv("VAR")**

**Strexpend( $x$ )**

## Parallel evaluation

These functions evaluate their arguments in parallel (pthreads or MPI); args. must not access global variables and must be free of side effects. Enabled if threading engine is not *single* in gp header.

evaluate  $f$  on  $x[1], \dots, x[n]$

evaluate closures  $f[1], \dots, f[n]$

as **select**

as **sum**

as **vector**

eval  $f$  for  $i = a, \dots, b$

... for  $p$  prime in  $[a, b]$

... multivariate

declare  $x$  as inline (allows to use as global)

stop inlining

**parapply( $f, x$ )**

**pareval( $f$ )**

**parselect( $f, A, \{flag\}$ )**

**parsum( $i = a, b, expr, \{x\}$ )**

**parvector( $n, i, \{expr\}$ )**

**parfor( $i = a, \{b\}, f, \{r\}, \{f_2\}$ )**

**parforprime( $p = a, \{b\}, f, \{r\}, \{f_2\}$ )**

**parforvec( $X = v, f, \{r\}, \{f_2\}, \{flag\}$ )**

**inline( $x$ )**

**uninline()**

## Linear Algebra

dimensions of matrix  $x$

multiply two matrices

... assuming result is diagonal

concatenation of  $x$  and  $y$

extract components of  $x$

transpose of vector or matrix  $x$

adjoint of the matrix  $x$

eigenvectors/values of matrix  $x$

characteristic/minimal polynomial of  $x$

trace/determinant of matrix  $x$

permanent of matrix  $x$

Frobenius form of  $x$

QR decomposition

apply **matqr**'s transform to  $v$

## Constructors & Special Matrices

$\{g(x): x \in v \text{ s.t. } f(x)\}$

$\{x: x \in v \text{ s.t. } f(x)\}$

$\{g(x): x \in v\}$

row vec. of  $expr$  eval'ed at  $1 \leq i \leq n$

col. vec. of  $expr$  eval'ed at  $1 \leq i \leq n$

vector of small ints

$[c, c \cdot x, \dots, c \cdot x^n]$

matrix  $1 \leq i \leq m, 1 \leq j \leq n$

define matrix by blocks

diagonal matrix with diagonal  $x$

is  $x$  diagonal?

$x \cdot \text{matdiagonal}(d)$

$n \times n$  identity matrix

Hessenberg form of square matrix  $x$

$n \times n$  Hilbert matrix  $H_{ij} = (i + j - 1)^{-1}$

$n \times n$  Pascal triangle

companion matrix to polynomial  $x$

Sylvester matrix of  $x$

**matsize( $x$ )**

$x * y$

**matmultodiagonal( $x, y$ )**

**concat( $x, \{y\}$ )**

**vecextract( $x, y, \{z\}$ )**

**mattranspose( $x$ )** or  $x \sim$

**matadjoint( $x$ )**

**mateigen( $x$ )**

**charpoly( $x$ ), minpoly**

**trace( $x$ ), matdet**

**matpermanent( $x$ )**

**matfrobenius( $x$ )**

**matqr( $x$ )**

**mathouseholder( $Q, v$ )**

**[g(x) | x <- v, f(x)]**

**[x | x <- v, f(x)]**

**[g(x) | x <- v]**

**vector( $n, \{i\}, \{expr\}$ )**

**vectorv( $n, \{i\}, \{expr\}$ )**

**vectorsmall( $n, \{i\}, \{expr\}$ )**

**powers( $x, n, \{c = 1\}$ )**

**matrix( $m, n, \{i\}, \{j\}, \{expr\}$ )**

**matconcat( $B$ )**

**matdiagonal( $x$ )**

**matisdiagonal( $x$ )**

**matmuldiagonal( $x, d$ )**

**matid( $n$ )**

**mathess( $x$ )**

**mathilbert( $n$ )**

**matpascal( $n - 1$ )**

**matcompanion( $x$ )**

**polsylvestermatrix( $x$ )**

## Gaussian elimination

kernel of matrix  $x$

intersection of column spaces of  $x$  and  $y$

solve  $MX = B$  ( $M$  invertible)

one sol of  $M * X = B$

basis for image of matrix  $x$

columns of  $x$  *not* in **matimage**

supplement columns of  $x$  to get basis

rows, cols to extract invertible matrix

rank of the matrix  $x$

solve  $MX = B \bmod D$

image mod  $D$

kernel mod  $D$

inverse mod  $D$

determinant mod  $D$

**matker( $x, \{flag\}$ )**

**matintersect( $x, y$ )**

**matsolve( $M, B$ )**

**matinverseimage( $M, B$ )**

**matimage( $x$ )**

**matimagecompl( $x$ )**

**mataugment( $x$ )**

**matindexrank( $x$ )**

**matrank( $x$ )**

**matmodmod( $M, D, B$ )**

**matmodmod( $M, D$ )**

**matkermod( $M, D$ )**

**matinvmod( $M, D$ )**

**matdetmod( $M, D$ )**

## Lattices & Quadratic Forms

### Quadratic forms

evaluate  ${}^t x Q y$

evaluate  ${}^t x Q x$

signature of quad form  ${}^t y * x * y$

decomp into squares of  ${}^t y * x * y$

eigenvalues/vectors for real symmetric  $x$

**qfeval( $\{Q = id\}, x, y$ )**

**qfeval( $\{Q = id\}, x$ )**

**qfsign( $x$ )**

**qfgaussred( $x$ )**

**qfjacobi( $x$ )**

### HNF and SNF

upper triangular Hermite Normal Form

HNF of  $x$  where  $d$  is a multiple of  $\det(x)$

multiple of  $\det(x)$

HNF of  $(x \mid \text{diagonal}(D))$

elementary divisors of  $x$

elementary divisors of  $\mathbf{Z}[a]/(f'(a))$

integer kernel of  $x$

**Z**-module  $\leftrightarrow$  **Q**-vector space

**mathnf( $x$ )**

**mathnfmod( $x, d$ )**

**matdetint( $x$ )**

**mathnfmodid( $x, D$ )**

**matsnf( $x$ )**

**poldiscreduced( $f$ )**

**matkerint( $x$ )**

**matrixqz( $x, p$ )**

### Lattices

LLL-algorithm applied to columns of  $x$

... for Gram matrix of lattice

find up to  $m$  sols of **qfnorm**( $x, y$ )  $\leq b$

$v, v[i] :=$  number of  $y$  s.t. **qfnorm**( $x, y$ ) =  $i$

perfection rank of  $x$

find isomorphism between  $q$  and  $Q$

precompute for isomorphism test with  $q$

automorphism group of  $q$

convert **qfauto** for GAP/Magma

orbits of  $V$  under  $G \subset \text{GL}(V)$

**qflll( $x, \{flag\}$ )**

**qflllgram( $x, \{flag\}$ )**

**qfminim( $x, b, m$ )**

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## Coefficients, variables and basic operators

degree of $f$	<code>poldegree(f)</code>
coef. of degree $n$ of $f$ , leading coef.	<code>polcoef(f,n), pollead</code>
main variable / all variables in $f$	<code>variable(f), variables(f)</code>
replace $x$ by $y$ in $f$	<code>subst(f,x,y)</code>
evaluate $f$ replacing vars by their value	<code>eval(f)</code>
replace polynomial expr. $T(x)$ by $y$ in $f$	<code>substpol(f,T,y)</code>
replace $x_1, \dots, x_n$ by $y_1, \dots, y_n$ in $f$	<code>substvec(f,x,y)</code>
reciprocal polynomial $x^{\deg f} f(1/x)$	<code>polrecip(f)</code>
gcd of coefficients of $f$	<code>content(f)</code>
derivative of $f$ w.r.t. $x$	<code>deriv(f,{x})</code>
formal integral of $f$ w.r.t. $x$	<code>intformal(f,{x})</code>
formal sum of $f$ w.r.t. $x$	<code>sumformal(f,{x})</code>

## Constructors & Special Polynomials

interpolating pol. eval. at $a$	<code>polinterpolate(X,{Y},{a})</code>
$P_n, T_n/U_n, H_n$	<code>pollegendre, polchebyshev, polhermite</code>
$n$ -th cyclotomic polynomial $\Phi_n$	<code>polcyclo(n,{v})</code>
return $n$ if $f = \Phi_n$ , else 0	<code>poliscyclo(f)</code>
is $f$ a product of cyclotomic polynomials?	<code>poliscycloprod(f)</code>
Zagier's polynomial of index $(n,m)$	<code>polzagier(n,m)</code>

## Resultant, elimination

discriminant of polynomial $f$	<code>poldisc(f)</code>
find factors of <code>poldisc(f)</code>	<code>poldiscfactors(f)</code>
resultant $R = \text{Res}_v(f,g)$	<code>polresultant(f,g,{v})</code>
$[u,v,R], xu + yv = \text{Res}_v(f,g)$	<code>polresultantext(x,y,{v})</code>
solve Thue equation $f(x,y) = a$	<code>thue(t,a,{sol})</code>
initialize $t$ for Thue equation solver	<code>thueinit(f)</code>

## Roots and Factorization (Complex/Real)

complex roots of $f$	<code>polroots(f)</code>
bound complex roots of $f$	<code>polrootsbound(f)</code>
number of real roots of $f$ (in $[a,b]$ )	<code>polsturm(f,{[a,b]})</code>
real roots of $f$ (in $[a,b]$ )	<code>polrootsreal(f,{[a,b]})</code>
complex embeddings of $t_{\text{POLMOD}} z$	<code>conjvec(z)</code>

## Roots and Factorization (Finite fields)

factor $f \bmod p$ , roots	<code>factormod(f,p), polrootsmod</code>
factor $f$ over $\mathbf{F}_p[x]/(T)$ , roots	<code>factormod(f,[T,p]), polrootsmod</code>
squarefree factorization of $f$ in $\mathbf{F}_q[x]$	<code>factormodSQF(f,{D})</code>
distinct degree factorization of $f$ in $\mathbf{F}_q[x]$	<code>factormodDDF(f,{D})</code>

## Roots and Factorization ( $p$ -adic fields)

factor $f$ over $\mathbf{Q}_p$ , roots	<code>factorpadic(f,p,r), polrootspadic</code>
$p$ -adic root of $f$ congruent to $a \bmod p$	<code>padicappr(f,a)</code>
Newton polygon of $f$ for prime $p$	<code>newtonpoly(f,p)</code>
Hensel lift $A/\text{lc}(A) = \prod_i B[i] \bmod p^e$	<code>polhensellift(A,B,p,e)</code>
extensions of $\mathbf{Q}_p$ of degree $N$	<code>padicfields(p,N)</code>

## Roots and Factorization (Miscellaneous)

symmetric powers of roots of $f$ up to $n$	<code>polsym(f,n)</code>
Graeffe transform of $f, g(x^2) = f(x)f(-x)$	<code>polgraeffe(f)</code>
factor $f$ over coefficient field	<code>factor(f)</code>
cyclotomic factors of $f \in \mathbf{Q}[X]$	<code>polcyclofactors(f)</code>

## Finite Fields

A finite field is encoded by any element ( $t_{\text{FFELT}}$ ).	
find irreducible $T \in \mathbf{F}_p[x], \deg T = n$	<code>ffinit(p,n,{x})</code>
Create $t$ in $\mathbf{F}_q \simeq \mathbf{F}_p[t]/(T)$	<code>t = ffgen(T,'t)</code>
... indirectly, with implicit $T$	<code>t = ffgen(q,'t); T = t.mod</code>
map $m$ from $\mathbf{F}_q \ni a$ to $\mathbf{F}_{q^k} \ni b$	<code>m = ffmbed(a,b)</code>
build $K$ from $\mathbf{F}_q[x]/(P)$ extending $\mathbf{F}_q \ni a$ ,	<code>ffextend(a,P)</code>
evaluate map $m$ on $x$	<code>ffmap(m,x)</code>
inverse map of $m$	<code>ffinvmap(m)</code>
compose maps $m \circ n$	<code>ffcompomap(m,n)</code>
$F^n$ over $\mathbf{F}_q \ni a$	<code>fffrobenius(a,n)</code>
$\#\{\text{monic irred. } T \in \mathbf{F}_q[x], \deg T = n\}$	<code>ffnbirred(q,n)</code>

## Formal & $p$ -adic Series

truncate power series or $p$ -adic number	<code>truncate(x)</code>
valuation of $x$ at $p$	<code>valuation(x,p)</code>
<b>Dirichlet and Power Series</b>	
Taylor expansion around 0 of $f$ w.r.t. $x$	<code>taylor(f,x)</code>
Laurent series expansion around 0 up to $x^k$	<code>laurentseries(f,k)</code>
$\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$	<code>serconvol(a,b)</code>
$f = \sum a_k t^k$ from $\sum (a_k/k!) t^k$	<code>serlaplace(f)</code>
reverse power series $F$ so $F(f(x)) = x$	<code>serreverse(f)</code>
remove terms of degree $< n$ in $f$	<code>serchop(f,n)</code>
Dirichlet series multiplication / division	<code>dirmul, dirdiv(x,y)</code>
Dirichlet Euler product ( $b$ terms)	<code>direuler(p=a,b,expr)</code>

## Transcendental and $p$ -adic Functions

real, imaginary part of $x$	<code>real(x), imag(x)</code>
absolute value, argument of $x$	<code>abs(x), arg(x)</code>
square/nth root of $x$	<code>sqrt(x), sqrtn(x,n,{&amp;z})</code>
trig functions	<code>sin, cos, tan, cotan, sinc</code>
inverse trig functions	<code>asin, acos, atan</code>
hyperbolic functions	<code>sinh, cosh, tanh, cotanh</code>
inverse hyperbolic functions	<code>asinh, acosh, atanh</code>
$\log(x), \log(1+x), e^x, e^x - 1$	<code>log, log1p, exp, expm1</code>
Euler $\Gamma$ function, $\log \Gamma, \Gamma'/\Gamma$	<code>gamma, lngamma, psi</code>
half-integer gamma function $\Gamma(n+1/2)$	<code>gammah(n)</code>
Riemann's zeta $\zeta(s) = \sum n^{-s}$	<code>zeta(s)</code>
Hurwitz's $\zeta(s,x) = \sum (n+x)^{-s}$	<code>zetahurwitz(s,x)</code>
multiple zeta value (MZV), $\zeta(s_1, \dots, s_k)$	<code>zetamult(s,{T})</code>
... init $T$ for MZV with $s_1 + \dots + s_k \leq w$	<code>zetamultinit(w)</code>
all MZVs for all weights $\sum s_i \leq n$	<code>zetamultall(n)</code>
convert MZV id to $[s_1, \dots, s_k]$	<code>zetamultconvert(f,{flag})</code>
incomplete $\Gamma$ function ( $y = \Gamma(s)$ )	<code>incgam(s,x,{y})</code>
complementary incomplete $\Gamma$	<code>incgamc(s,x)</code>
$\int_x^\infty e^{-t} dt/t, (2/\sqrt{\pi}) \int_x^\infty e^{-t^2} dt$	<code>eint1, erfc</code>
dilogarithm of $x$	<code>dilog(x)</code>
$m$ -th polylogarithm of $x$	<code>polylog(m,x,{flag})</code>
$U$ -confluent hypergeometric function	<code>hyperu(a,b,u)</code>
Bessel $J_n(x), J_{n+1/2}(x)$	<code>besselj(n,x), besseljh(n,x)</code>
Bessel $I_\nu, K_\nu, H_\nu^1, H_\nu^2, N_\nu$	<code>(bessel)i,k,h1,h2,n</code>
Lambert $W: x$ s.t. $xe^x = y$	<code>lambertw(y)</code>
Teichmuller character of $p$ -adic $x$	<code>teichmuller(x)</code>

## Iterations, Sums & Products

### Numerical integration for meromorphic functions

Behaviour at endpoint for Double Exponential (DE) methods: either a scalar ( $a \in \mathbf{C}$ , regular) or $\pm\infty$ (decreasing at least as $x^{-2}$ ) or	
$(x-a)^{-\alpha}$ singularity	<code>[a,a]</code>
exponential decrease $e^{-\alpha x }$	<code>[<math>\pm\infty, \alpha</math>], <math>\alpha &gt; 0</math></code>
slow decrease $ x ^\alpha$	<code>... <math>\alpha &lt; -1</math></code>
oscillating as $\cos(kx)$	<code><math>\alpha = k\mathbf{I}, k &gt; 0</math></code>
oscillating as $\sin(kx)$	<code><math>\alpha = -k\mathbf{I}, k &gt; 0</math></code>
numerical integration	<code>intnum(<math>x=a,b,f,{T}</math>)</code>
weights $T$ for intnum	<code>intnuminit(<math>a,b,K,{m}</math>)</code>
weights $T$ incl. kernel $K$	<code>intfuncinit(<math>a,b,K,{m}</math>)</code>
integrate $(2i\pi)^{-1} f$ on circle $ z-a  = R$	<code>intcirc(<math>x=a,R,f,{T}</math>)</code>

### Other integration methods

$n$ -point Gauss-Legendre	<code>intnumgauss(<math>x=a,b,f,{n}</math>)</code>
weights for $n$ -point Gauss-Legendre	<code>intnumgaussinit(<math>\{n\}</math>)</code>
Romberg integration (low accuracy)	<code>intnumromb(<math>x=a,b,f,{flag}</math>)</code>

### Numerical summation

sum of series $f(n), n \geq a$ (low accuracy)	<code>suminf(<math>n=a,expr</math>)</code>
sum of alternating/positive series	<code>sumalt, sumpos</code>
sum of series using Euler-Maclaurin	<code>sumnum(<math>n=a,f,{T}</math>)</code>
$\sum_{n \geq a} F(n)$ , $F$ rational function	<code>sumnumrat(<math>F,a</math>)</code>
$\dots \sum_{n \geq a} (-1)^n F(n)$	<code>sumaltrat(<math>F,a</math>)</code>
$\dots \sum_{p \geq a} F(p^s)$	<code>sumeulerrat(<math>F,\{s=1\},\{a=2\}</math>)</code>
weights for sumnum, $a$ as in DE	<code>sumnuminit(<math>\{\infty,a\}</math>)</code>
sum of series by Monien summation	<code>sumnummonien(<math>n=a,f,{T}</math>)</code>
weights for sumnummonien	<code>sumnummonieninit(<math>\{\infty,a\}</math>)</code>
sum of series using Abel-Plana	<code>sumnumap(<math>n=a,f,{T}</math>)</code>
weights for sumnumap, $a$ as in DE	<code>sumnumapinit(<math>\{\infty,a\}</math>)</code>
sum of series using Lagrange	<code>sumnumlagrange(<math>n=a,f,{T}</math>)</code>
weights for sumnumlagrange	<code>sumnumlagrangeinit</code>

### Products

product $a \leq X \leq b$ , initialized at $x$	<code>prod(<math>X=a,b,expr,\{x\}</math>)</code>
product over primes $a \leq X \leq b$	<code>prodeuler(<math>X=a,b,expr</math>)</code>
infinite product $a \leq X \leq \infty$	<code>prodinfn(<math>X=a,expr</math>)</code>
$\prod_{n \geq a} F(n)$ , $F$ rational function	<code>prodnumrat(<math>F,a</math>)</code>
$\dots \prod_{p \geq a} F(p^s)$	<code>prodeulerrat(<math>F,\{s=1\},\{a=2\}</math>)</code>

### Other numerical methods

real root of $f$ in $[a,b]$ ; bracketed root	<code>solve(<math>X=a,b,f</math>)</code>
... by interval splitting	<code>solvestep(<math>X=a,b,f,{flag=0}</math>)</code>
limit of $f(t), t \rightarrow \infty$	<code>limitnum(f,{k},{alpha})</code>
asymptotic expansion of $f$ at $\infty$	<code>asypnum(f,{k},{alpha})</code>
numerical derivation w.r.t $x: f'(a)$	<code>derivnum(<math>x=a,f</math>)</code>
evaluate continued fraction $F$ at $t$	<code>contfraceval(<math>F,t,\{L\}</math>)</code>
power series to cont. fraction ( $L$ terms)	<code>confracinit(<math>S,\{L\}</math>)</code>
Padé approximant (deg. denom. $\leq B$ )	<code>bestapprPade(<math>S,\{B\}</math>)</code>

Elementary Arithmetic Functions

vector of binary digits of $ x $	<code>binary(<math>x</math>)</code>
bit number $n$ of integer $x$	<code>bittest(<math>x, n</math>)</code>
Hamming weight of integer $x$	<code>hammingweight(<math>x</math>)</code>
digits of integer $x$ in base $B$	<code>digits(<math>x, \{B = 10\}</math>)</code>
sum of digits of integer $x$ in base $B$	<code>sumdigits(<math>x, \{B = 10\}</math>)</code>
integer from digits	<code>fromdigits(<math>v, \{B = 10\}</math>)</code>
ceiling/floor/fractional part	<code>ceil, floor, frac</code>
round $x$ to nearest integer	<code>round(<math>x, \{\&amp;e\}</math>)</code>
truncate $x$	<code>truncate(<math>x, \{\&amp;e\}</math>)</code>
gcd/LCM of $x$ and $y$	<code>gcd(<math>x, y</math>), lcm(<math>x, y</math>)</code>
gcd of entries of a vector/matrix	<code>content(<math>x</math>)</code>
<b>Primes and Factorization</b>	
extra prime table	<code>addprimes()</code>
add primes in $v$ to prime table	<code>addprimes(<math>v</math>)</code>
remove primes from prime table	<code>removeprimes(<math>v</math>)</code>
Chebyshev $\pi(x)$ , $n$ -th prime $p_n$	<code>primepi(<math>x</math>), prime(<math>n</math>)</code>
vector of first $n$ primes	<code>primes(<math>n</math>)</code>
smallest prime $\geq x$	<code>nextprime(<math>x</math>)</code>
largest prime $\leq x$	<code>preprime(<math>x</math>)</code>
factorization of $x$	<code>factor(<math>x, \{lim\}</math>)</code>
...selecting specific algorithms	<code>factorint(<math>x, \{flag = 0\}</math>)</code>
$n = df^2$ , $d$ squarefree/fundamental	<code>core(<math>n, \{fl\}</math>), coredisc</code>
certificate for (prime) $N$	<code>primecert(<math>N</math>)</code>
verifies a certificate $c$	<code>primecertisvalid(<math>c</math>)</code>
convert certificate to Magma/PRIMO	<code>primecertexport</code>
recover $x$ from its factorization	<code>factorback(<math>f, \{e\}</math>)</code>
$x \in \mathbf{Z}$ , $ x  \leq X$ , $\gcd(N, P(x)) \geq N$	<code>zncoppersmith(<math>P, N, X, \{B\}</math>)</code>
divisors of $N$ in residue class $r \bmod s$	<code>divisorslenstra(<math>N, r, s</math>)</code>
<b>Divisors and multiplicative functions</b>	
number of prime divisors $\omega(n)$ / $\Omega(n)$	<code>omega(<math>n</math>), bigomega</code>
divisors of $n$ / number of divisors $\tau(n)$	<code>divisors(<math>n</math>), numdiv</code>
sum of ( $k$ -th powers of) divisors of $n$	<code>sigma(<math>n, \{k\}</math>)</code>
Möbius $\mu$ -function	<code>moebius(<math>x</math>)</code>
Ramanujan's $\tau$ -function	<code>ramanujantau(<math>x</math>)</code>
<b>Combinatorics</b>	
factorial of $x$	<code><math>x!</math> or factorial(<math>x</math>)</code>
binomial coefficient $\binom{x}{k}$	<code>binomial(<math>x, \{k\}</math>)</code>
Bernoulli number $B_n$ as real/rational	<code>bernreal(<math>n</math>), bernfrac</code>
Bernoulli polynomial $B_n(x)$	<code>bernpol(<math>n, \{x\}</math>)</code>
$n$ -th Fibonacci number	<code>fibonacci(<math>n</math>)</code>
Stirling numbers $s(n, k)$ and $S(n, k)$	<code>stirling(<math>n, k, \{flag\}</math>)</code>
number of partitions of $n$	<code>numbpart(<math>n</math>)</code>
$k$ -th permutation on $n$ letters	<code>numtoperm(<math>n, k</math>)</code>
convert permutation to $(n, k)$ form	<code>permtotnum(<math>v</math>)</code>
order of permutation $p$	<code>permorder(<math>p</math>)</code>
signature of permutation $p$	<code>permsign(<math>p</math>)</code>
<b>Multiplicative groups <math>(\mathbf{Z}/N\mathbf{Z})^*</math>, <math>\mathbf{F}_q^*</math></b>	
Euler $\phi$ -function	<code>eulerphi(<math>x</math>)</code>
multiplicative order of $x$ (divides $\phi$ )	<code>znorder(<math>x, \{o\}</math>), fforder</code>
primitive root mod $q$ / $x$ .mod	<code>znprimroot(<math>q</math>), ffprimroot(<math>x</math>)</code>
structure of $(\mathbf{Z}/n\mathbf{Z})^*$	<code>znstar(<math>n</math>)</code>
discrete logarithm of $x$ in base $g$	<code>znlog(<math>x, g, \{o\}</math>), fflag</code>
Kronecker-Legendre symbol $(\frac{x}{y})$	<code>kronecker(<math>x, y</math>)</code>
quadratic Hilbert symbol (at $p$ )	<code>hilbert(<math>x, y, \{p\}</math>)</code>

<b>Miscellaneous</b>	
integer square / $n$ -th root of $x$	<code>sqrntint(<math>x</math>), sqrtnint(<math>x, n</math>)</code>
largest integer $e$ s.t. $b^e \leq b$ , $e = \lfloor \log_b(x) \rfloor$	<code>logint(<math>x, b, \{\&amp;z\}</math>)</code>
CRT: solve $z \equiv x$ and $z \equiv y$	<code>chinese(<math>x, y</math>)</code>
minimal $u, v$ so $xu + yv = \gcd(x, y)$	<code>gcdext(<math>x, y</math>)</code>
continued fraction of $x$	<code>contfrac(<math>x, \{b\}, \{lmax\}</math>)</code>
last convergent of continued fraction $x$	<code>confracpnqn(<math>x</math>)</code>
rational approximation to $x$ (den. $\leq B$ )	<code>bestappr(<math>x, \{B\}k</math>)</code>
recognize $x \in \mathbf{C}$ as polmod mod $T \in \mathbf{Z}[X]$	<code>bestapprnf(<math>x, T</math>)</code>

Characters

Let $cyc = [d_1, \dots, d_k]$ represent an abelian group $G = \oplus (\mathbf{Z}/d_j\mathbf{Z}) \cdot g_j$ or any structure $G$ affording a .cyc method; e.g. <code>znstar(<math>q, 1</math>)</code> for Dirichlet characters. A character $\chi$ is coded by $[c_1, \dots, c_k]$ such that $\chi(g_j) = e(n_j/d_j)$ .	
$\chi \cdot \psi$ ; $\chi^{-1}$ ; $\chi \cdot \psi^{-1}$ ; $\chi^k$	<code>charmul, charconj, chardiv,, charpow</code>
order of $\chi$	<code>charorder(<math>cyc, \chi</math>)</code>
kernel of $\chi$	<code>charker(<math>cyc, \chi</math>)</code>
$\chi(x)$ , $G$ a GP group structure	<code>chareval(<math>G, \chi, x, \{z\}</math>)</code>
Galois orbits of characters	<code>chargalois(<math>G</math>)</code>

Dirichlet Characters

initialize $G = (\mathbf{Z}/q\mathbf{Z})^*$	<code>G = znstar(<math>q, 1</math>)</code>
convert datum $D$ to $[G, \chi]$	<code>znchar(<math>D</math>)</code>
is $\chi$ odd?	<code>zncharisodd(<math>G, \chi</math>)</code>
real $\chi \rightarrow$ Kronecker symbol $(D/.)$	<code>znchartokronecker(<math>G, \chi</math>)</code>
conductor of $\chi$	<code>zncharconductor(<math>G, \chi</math>)</code>
$[G_0, \chi_0]$ primitive attached to $\chi$	<code>znchartoprimitive(<math>G, \chi</math>)</code>
induce $\chi \in \hat{G}$ to $\mathbf{Z}/N\mathbf{Z}$	<code>zncharinduce(<math>G, \chi, N</math>)</code>
$\chi_p$	<code>znchardecompose(<math>G, \chi, p</math>)</code>
$\prod_p  (\mathbf{Q}, N) \chi_p$	<code>znchardecompose(<math>G, \chi, \mathbf{Q}</math>)</code>
complex Gauss sum $G_a(\chi)$	<code>znchargauss(<math>G, \chi</math>)</code>

Conrey labelling

Conrey label $m \in (\mathbf{Z}/q\mathbf{Z})^* \rightarrow$ character	<code>znconreychar(<math>G, m</math>)</code>
character $\rightarrow$ Conrey label	<code>znconreyexp(<math>G, \chi</math>)</code>
log on Conrey generators	<code>znconreylog(<math>G, m</math>)</code>
conductor of $\chi$ ( $\chi_0$ primitive)	<code>znconreyconductor(<math>G, \chi, \{\chi_0\}</math>)</code>

True-False Tests

is $x$ the disc. of a quadratic field?	<code>isfundamental(<math>x</math>)</code>
is $x$ a prime?	<code>isprime(<math>x</math>)</code>
is $x$ a strong pseudo-prime?	<code>ispseudoprime(<math>x</math>)</code>
is $x$ square-free?	<code>issquarefree(<math>x</math>)</code>
is $x$ a square?	<code>issquare(<math>x, \{\&amp;n\}</math>)</code>
is $x$ a perfect power?	<code>ispower(<math>x, \{k\}, \{\&amp;n\}</math>)</code>
is $x$ a perfect power of a prime? ( $x = p^n$ )	<code>isprimepower(<math>x, \&amp;n\}</math>)</code>
... of a pseudoprime?	<code>ispseudoprimepower(<math>x, \&amp;n\}</math>)</code>
is $x$ powerful?	<code>ispowerful(<math>x</math>)</code>
is $x$ a totient? ( $x = \varphi(n)$ )	<code>istotient(<math>x, \{\&amp;n\}</math>)</code>
is $x$ a polygonal number? ( $x = P(s, n)$ )	<code>ispolygonal(<math>x, s, \{\&amp;n\}</math>)</code>
is $pol$ irreducible?	<code>polisirreducible(<math>pol</math>)</code>

Graphic Functions

crude graph of $expr$ between $a$ and $b$	<code>plot(<math>X = a, b, expr</math>)</code>
<b>High-resolution plot</b> (immediate plot)	
plot $expr$ between $a$ and $b$	<code>plotth(<math>X = a, b, expr, \{flag\}, \{n\}</math>)</code>
plot points given by lists $lx, ly$	<code>plotthraw(<math>lx, ly, \{flag\}</math>)</code>
terminal dimensions	<code>plotsizes()</code>

Rectwindow functions

init window $w$ , with size $x, y$	<code>plotinit(<math>w, x, y</math>)</code>
erase window $w$	<code>plotkill(<math>w</math>)</code>
copy $w$ to $w_2$ with offset $(dx, dy)$	<code>plotcopy(<math>w, w_2, dx, dy</math>)</code>
clips contents of $w$	<code>plotclip(<math>w</math>)</code>
scale coordinates in $w$	<code>plotscale(<math>w, x_1, x_2, y_1, y_2</math>)</code>
plot $in\ w$	<code>plotrecth(<math>w, X = a, b, expr, \{flag\}, \{n\}</math>)</code>
plot $throw\ in\ w$	<code>plotrecththrow(<math>w, data, \{flag\}</math>)</code>
draw window $w_1$ at $(x_1, y_1), \dots$	<code>plotdraw(<math>[[w_1, x_1, y_1], \dots]</math>)</code>

Low-level Rectwindow Functions

set current drawing color in $w$ to $c$	<code>plotcolor(<math>w, c</math>)</code>
current position of cursor in $w$	<code>plotcursor(<math>w</math>)</code>
write $s$ at cursor's position	<code>plotstring(<math>w, s</math>)</code>
move cursor to $(x, y)$	<code>plotmove(<math>w, x, y</math>)</code>
move cursor to $(x + dx, y + dy)$	<code>plotrmove(<math>w, dx, dy</math>)</code>
draw a box to $(x_2, y_2)$	<code>plotbox(<math>w, x_2, y_2</math>)</code>
draw a box to $(x + dx, y + dy)$	<code>plotrbox(<math>w, dx, dy</math>)</code>
draw polygon	<code>plotlines(<math>w, lx, ly, \{flag\}</math>)</code>
draw points	<code>plotpoints(<math>w, lx, ly</math>)</code>
draw line to $(x + dx, y + dy)$	<code>plotrline(<math>w, dx, dy</math>)</code>
draw point $(x + dx, y + dy)$	<code>plotrpoint(<math>w, dx, dy</math>)</code>
draw point $(x + dx, y + dy)$	<code>plotrpoint(<math>w, dx, dy</math>)</code>

Convert to Postscript or Scalable Vector Graphics

The format $f$ is either "ps" or "svg".	
as plot	<code>plotlexport(<math>f, X = a, b, expr, \{flag\}, \{n\}</math>)</code>
as plotdraw	<code>plotthrawlexport(<math>f, lx, ly, \{flag\}</math>)</code>
as plotdraw	<code>plotlexport(<math>f, [[w_1, x_1, y_1], \dots]</math>)</code>

Based on an earlier version by Joseph H. Silverman  
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